Inverse Design Criteria for Supersonic Internally Pressurized Polymer Domes

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Inverse design methodology is used to relate supersonic boundary conditions to the material properties of an internally pressurized dome. Thermal and mechanical properties of a chemically stable polymer are related to internally pressurized hemispheric dome design. The proposed approach is used to predict material specifications for thermal conductivity and creep compliance moduli as a function of dome thickness and time to failure at Mach 3 flow at an altitude of 5000 ft. The procedure also is used for determining dome thickness requirements as a function of mission lifetime.

Nomenclature

a = speed of sound, fps
a_T = temperature shift factor
C_{fe}, C_{fi} = local compressible and incompressible skin-friction coefficient
$C_p, C_{p\max}$ = aerodynamic pressure and maximum pressure coefficient
c_p = specific heat at constant pressure, Btu/lbm − °R
E = Young's modulus, lbf/ft^2
h = steady-state heat convection coefficient, Btu/ft^2-hr-°R
h_d = dissipation steady-state heat convection coefficient, Btu/ft^2-hr-°R
$J(t)$ = creep compliance, ft^2/lbf
k_a, k_p = thermal conductivity of air and polymer, Btu/ft-hr-°R
$M, N_B, N_{nu}, N_{pr}, N_{re}$ = Mach, Biot, Nusselt, Prandtl, and Reynolds numbers
p = pressure, lbf/ft^2
q_a = heat flux available to dome, Btu/ft^2-hr
R_0 = dome hemispheric radius, ft
r = recovery factor
s, s_c = wetted surface distance, from apex of cone, ft
T, T_p, T_r = temperature, polymer, recovery, °F, or °R
t = time, hr, min, or sec
U = velocity, fps
δ = dome thickness, ft
ϵ_{ij} = dome strains, in./in.
θ = angle between the flight direction and the radius vector, deg
ξ_n, ξ_l = environmental normal pressure and shear stress on dome, lbf/ft^2
ρ, ρ_p = density, of polymer, lbm/ft^3
σ_{ij} = membrane loadings, lbf/ft
ν = Poisson's ratio
τ' = retardation time, hr

Subscripts

e = denotes conditions at outer edge of boundary layer
l = denotes laminar conditions
sa, sd = denotes conditions in air adjacent to dome, in dome adjacent to air
t = denotes turbulent conditions
ϕ = denotes stagnation conditions

Superscript

$*$ = denotes reference temperature conditions

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Introduction

INTEREST exists to determine the feasibility of using a thin, pressurized, high-temperature-resistant polymer missile dome for supersonic flight. Internal pressure minimizes the effective flight loading at regions of highest thermal loading, thus extending the material/mission time to failure. With inverse design methodology[1,2] it is possible to quantify the thermal and mechanical moduli specifications required for candidate high-temperature-resistant polymers given the mission design requirements.

In this study the thermal and mechanical boundary conditions are predicted by use of assumptions and semiempirical relationships. The resulting prediction of the boundary layer then is related to polymer behavior, by using heat transfer and viscoelasticity to obtain the required thermal and mechanical properties. Design criteria are developed based on a stress balance in the stagnation region and on the melt temperature at the inner boundary of the dome. The time to failure as a function of dome thickness δ, thermal conductivity k_p, and creep compliance $J(t)$ is predicted. Results are presented for a hemispheric polymer dome subjected to Mach 3 flow and 5000-ft-alt-conditions.

Thermomechanics of Supersonic Flight

The distinguishing features of supersonic flight of a blunt-nosed body of revolution are well-known. Although numerous rigorous analytical descriptions of the boundary layer have been proposed, the materials engineer must choose a relatively simple analysis selectively if material specifications are to be quantified practically.

Thermal Boundary Conditions

Assume that the dome surface temperature T_{sd} is equal to the temperature of the adjacent air T_{sa}. Then the boundary-layer dissipation heat-transfer coefficient is defined as

$$h_d = q_a / (T_{sd} - T_r) \tag{1}$$

where T_r is the dome recovery temperature and q_a is the heat flux available to heat the dome.

The classical heat transfer coefficient h, which is related to h_d, is characterized by the temperature-referenced Nusselt number

$$N^*_{nu} = hs/k^*_a \tag{2}$$

where s is the wetted distance from the apex of the pressurized dome and k^*_a is the temperature-referenced thermal conductivity of air. Estimates for N^*_{nu} can be made from flat-plate data[3] given the referenced temperature[4] T^*, here defined by

$$T^* - T_e = 0.5(T_{sd} - T_e) + 0.22(T_r - T_e) \quad (3)$$

where T_e is the temperature at the outer edge of the boundary layer. Note that Eq. (3) differs from that in Ref. 4 by substitution of T_{sd} for T_{su}.

The relationship[5] between h and h_d can be approximated by

$$h/h_d = 1 - [2.8\times 10^{-5} U_e^2/c_p(T_{sd} - T_e)] \quad (4)$$

where U_e is the local velocity at the outer boundary-layer edge and c_p is the specific heat of the ambient air at constant pressure.

The method of Fay and Riddell[6] is used to estimate the heat available to the dome at the stagnation region $(q_a)_\phi$. The thermal heat potential available to other parts of a hemisphere can be obtained using Lees' prediction,[7] which is valid for laminar flow. As a first approximation, it can be assumed that laminar flow exists over the entire surface of the pressurized dome. The consequence is to predict a lower q_a distribution than may actually exist. This approximation must be tested. If adequate, then q_a in the turbulent flow region must be quantitized.

The wetted surface of a hemisphere, bounded by the angle θ between the flight direction and the radius vector R_0, can be related to the wetted surface of a cone, which in turn can be related to that of a plate. The wetted edge of this cone is tangent to the edge of the wetted surface of the hemisphere. The wetted surface length s_c of the tangent cone is related to θ by

$$s_c = R_0 \tan\theta \quad (5)$$

For equal boundary-layer edge flow conditions, the heating rate at a point on a cone is equal to that at three times the distance from the leading edge of a flat plate. Thus, by treating a point on the hemisphere as belonging to a cone defined by the tangent, plate heat boundary-layer data can be used to estimate the heat boundary layer on a hemisphere. Substituting Eq. (1) into Eq. (4), and utilizing Eqs. (2) and (5), leads to an estimate of the available heat potential in the boundary layer with respect to T_{sd} as

$$q_a = (3N^*_{nu}k^*_a/R_0 \tan\theta)(T_{sd} - T_r)\{1 - 2\times 10^{-5} r\, U_e^2[c_p(T_{sd} - T_e)]^{-1}\}^{-1} \quad (6)$$

where r is the recovery factor associated with T_r. The value for N^*_{nu} applicable for a plate can be obtained near the initiation of transition from laminar flow,[3] with the assumption that the temperature-referenced Reynolds's number $N^*_{re} = 10^5$. q_a obtained in this manner can be equated to q_a from Lees' equation[7] in order to determine θ where laminar flow ends. The point of full turbulence can be assumed to occur where $N^*_{re} = 10^6$ on the equivalent cone. For this point, θ can be obtained in a similar way; q_a, in the region of full turbulence, is obtained from Eq. (6) by use of the appropriate N^*_{nu} (Ref. 3).

Mechanical Boundary Conditions

Two predominant mechanical loadings on a pressurized dome are normal loadings ξ_n (which are large at the blunt end of a dome and decrease in magnitude downstream), and shear loadings ξ_l (which are zero at the stagnation point and increase in magnitude downstream). The local pressure p on the surface of a body of revolution can be computed[8] given the aerodynamic pressure coefficient C_p. For a hemisphere-cylinder (valid for $0.8 < M < 2$) Andrews' relationship[9] can be used to compute C_p as a function of $(90-\theta)$, $C_{p_{max}}$ and Mach number M. With the Rankine-Hugoniot relation,[8] it follows that the pressure distribution over the dome can be computed readily.

The local skin-friction coefficient at the boundary layer to dome interface is

$$C_{fe} = 2\xi_l/\rho_e U_e^2 \quad (7)$$

where ρ_e is the air density at the outer edge of the boundary layer. The corresponding incompressible local skin-friction coefficient C_{fi}, is a function of N_{re}. This relationship, as proposed by Falkner,[10] is modified by replacing N_{re} by N^*_{re} to give

$$C_{fi} = 0.0262\, N^{*\,-1/7}_{re} \quad (8)$$

The relationship between C_{fe} and C_{fi} is given[10] by

$$C_{fe}/C_{fi} = \{1 + r[(\gamma-1)/4]\ M_e^2\}^{-0.714} \quad (9)$$

Solving Eq. (6) for N^*_{nu}, using the relationship $N^*_{re} = f(N^*_{nu}, N^*_{pr})$ for plates,[3] and assuming that $(N^*_{pr})_{ave} = 0.84$ results in

$$N^*_{re} = \{1.065\, q_a R_0 \tan\theta/k^*_a(T_{sd} - T_r)[1 - 2\times 10^{-5} r U_e^2/c_p(T_{sd} - T_e)]\}^2 \quad (10)$$

Substituting Eq. (10) into Eq. (8) enables C_{fi} to be computed. If follows that ξ_l in Eq. (7) can be obtained by use of the results from Eq. (9).

Example Computations of Boundary Conditions

The prediction of the boundary conditions on the surface of a hemisphere-cylinder ($R_0 = 0.208$ ft) traveling at Mach 3 at an altitude of 5000 ft is given in Table 1. Pertinent environment and physical properties for a hot-air atmosphere needed in the computations are found in Refs. 3 and 11. Boundary conditions at the stagnation region are required, using the Rankine-Hugoniot relations, the ideal gas law, and Sutherland's equation[12] for viscosity. In order to typify a high-heat-resistant polymer, $T_{sd} = 1080°$R.

The distributed available heat potential is computed next. Andrew's relationship[9] is used to obtain both $C_{p_{max}} = 1.75$ and C_p as a function of θ (see Table 1, lines 1-2). The computed $(p_{sd})_{alt} = f(90-\theta)$ values are given in Table 1, line 3. Isentopic flow relationships are used to obtain T_e and U_e^2, which are given in Table 1, lines 5-6. The local values for T_r are obtained assuming $r = 0.84$; T_r, together with a_e and M_e, are tabulated in Table 1, lines 7-9; T^* is computed using Eq. (3), given T_e and T_r; k^* is obtained from Ref. 12, given T^*. Both k^*_a and T^* are listed in Table 1, lines 10-11. Line 12 of Table 1 shows the computed turbulent flow value for $(q_a)_t$ as a function of θ. In order to find θ, at the transition from laminar to turbulent flow, first one must compute that the heat flow potential around the dome, assuming laminar flow everywhere. Lees' equation[7] is used to obtain $q_a/(q_a)_\phi$ (Table 1, line 13); q_a is given in Table 1, line 14 as a function of θ. The transition point at which laminar flow ends is found by equating $(q_a)_t$ to q_a. Transition is predicted at $\theta \sim 56°$.

One obtains ξ_l from Eqs. (10, 8, 9, and 7), in that order, using the data from Table 1; ξ_l is given in Table 1, line 15, as a function of θ. The maximum shear stress to be reacted by the pressurized dome is about 808 lbf/ft^2 (7 lbf/in.^{2}) at $\theta = 56°$. The maximum available heat flux potential and normal applied pressures are computed to be 1.7 Btu/ft^2-sec at $\theta = 0°$ and 21,400 lbf/ft^2 (153 lbf/in.^{2}) at $\theta = 90°$, respectively.

Polymer Viscoelastic Properties

For an elastic material, the membrane strains ϵ_{ij} are expressed in terms of the loadings σ_{ij}, δ, Young's modulus E,

Table 1 Example computations for boundary conditions on hemispheric dome

θ, deg	90	80	70	60	50	40	30	20	10	0
(C_p)	−0.011	+0.056	0.23	0.46	0.74	1.05	1.34	1.57	1.73	1.75
$(p_{sd})_{5,000}$lbf/ft^2	1,650	2,400	4,360	6,930	10,100	13,600	16,800	19,400	21,200	21,400
(p_{sd}/p_ϕ)	12.5	9.1	5.0	3.12	2.13	1.56	1.27	1.10	1.01	1.0
$(T_e)_{5,000}$, °R	740	809	961	1100	1230	1330	1420	1480	1510	1520
$(U_e^2)_{5,000}$, ft^2/sec^2 x 10^6	9.46	8.59	6.89	5.10	3.60	2.19	1.22	0.50	0.04	0
$(a_e)_{5,000}$, ft/sec	1,340	1,390	1,520	1,630	1,720	1,790	1,850	1,890	1,910	1,920
M_e	2.30	2.10	1.73	1.38	1.10	0.84	0.61	0.37	0.11	0
$(T_r)_{5,000}$, °R	1,400	1,410	1,450	1,460	1,470	1,480	1,570	1,570	1,540	1,520
T^*, °R	1,050	1,080	1,120	1,170	1,200	1,240	1,290	1,300	1,300	1,300
k_a^*, BTU/ft^2-sec-°R x 10^{-6}	7.3	7.3	7.5	7.8	8.1	8.2	8.4	8.6	8.6	8.6
$(q_a)_t$, BTU/ft^2-sec	(-)	(-)	-0.2	0.1	1.8	8.6	31.7	(-)	(-)	(-)
$q_a/(q_a)_\phi$	1	0.98	0.93	0.82	0.69	0.56	0.41	0.28	0.17	0.06
q_a, BTU/ft^2-sec	1.70	1.67	1.58	1.39	1.17	0.95	0.70	0.48	0.29	0.10
ξ_l, lbf/ft^2	0	255	497	687	808	770	660	416	50	0

and Poisson's ratio ν. The state of loading (lbf/ft) at a referenced temperature in a symmetrically deformed hemispherical membrane is given by

$$\sigma_{11}^* = R_0/\sin^2\theta\left(\int_0^\theta \xi_n \cos\theta \sin\theta d\theta - \int_0^\theta \xi_l \sin^2\theta d\theta\right) \qquad (11)$$

$$\sigma_{22}^* = \xi_n R_0 - \sigma_{11}^* \qquad (12)$$

An approximation of σ_{11}^* and σ_{22}^*, given the previous boundary conditions, is obtained next. The internal pressure should equal the maximum stagnation pressure to prevent buckling, i.e., $\xi_n = 21{,}400$ lbf/ft^2. Use the maximum value for shear stress, i.e., $\xi_l = 810$ lbf/ft^2. Assume that ξ_n and ξ_l are constant, and integrate Eq. (11). At the apex, $\sigma_{11}^* = \sigma_{22}^* = 2220$ lbf/ft. However, ξ_l and σ_{11}^* at other θ are less than 2220 lbf/ft, whereas σ_{22}^* is greater than 2220 lbf/ft. Near the base of the hemisphere $\sigma_{11}^* = 1590$ and $\sigma_{22}^* = 2860$ lbf/ft. The maximum dome loads for the given condition are about 3000 lbf/ft (21 lbf/in.). The calculated stress for 0.01−, 0.05−, or 0.10-in./thick domes are 2100, 420, or 210 lbf/in.^{2}, respectively. If the maximum strains are not to exceed 5%, then from the elastic relationships (assuming $\nu = 0.5$), $E > 20{,}400$, 11,200, and 2,040 lbf/in.^{2}, respectively. Polymeric materials having these elastic properties at ambient temperature are readily available. At higher temperatures, the polymer viscoelastic properties[12] must be considered. An engineering estimate of this viscoelastic behavior is obtained by assuming thermorheologically simple material behavior.[13]

The creep compliance for a membrane subjected to an initial step loading is given as

$$J(t) = \epsilon_{ii}^*(t)\delta/\sigma_{ii}^* \qquad (13)$$

where σ_{11}^* is constant, since the internal dome pressure can be regulated to the stagnation pressure; $\epsilon_{ii}^*(t)$ is a function of time at a given polymer temperature, e.g., T_{sd}; and $J(t)$ can be approximated as being equal to an inverse time-dependent

Table 2 Approximate tradeoff relationships for high-temperature-resistant polymers among k_p, δ, and t for $M = 3$ at 5000 ft alt

δ, in.	t, sec		
	k_p, Btu/ft-hr-°R		
	0.1	0.01	0.001
0.01	0.3	1.80	17
0.5	4.7	43	174
0.10	17.4	174	1740

E. The viscoelastic retardation time τ' depends on T_{sd}. If a polymer is in the glassy state and the operational time—i.e., the time of application of σ_{11}^*— is longer than τ', then $J(t)$ will increase significantly with times near τ', i.e., the structure will creep. On the other hand, if T_{sd} is high enough so that the polymer is the rubbery state, τ' is much smaller than the operational time, and the structure will not creep but will deform according to the higher value of $J(t)$. The temperature dependence of $J(t)$ is known given log a_T.[12] Although analytical expressions exist, practically, a_T should be obtained experimentally for candidate polymers.

An example computation is made for a mission time, which lasts t min. Assume that $\delta = 0.05$ in., and $\epsilon_{22} = 0.2$ in./in. at $t = \tau'$. Solving Eq. (13), one obtains $J(\tau') = 3.3 \times 10^{-6}$ in.^{2}/lbf. This value is reasonable for polymers at ambient temperature and nominal mission times. It is not known for how long this value can be attained for high-heat-resistant polymers at the higher temperatures, or whether low conductivity polymers will have comparable τ'. Pertinent experimental data are needed.

Polymer Thermal Properties

Thermal diffusion through a thin dome is evaluated using one-dimensional theory. If the thermal properties of the polymer are assumed constant, then the linear transient thermal diffusion analysis is simplified. Further assumptions are: 1) the initial dome temperature is T_r at $t=0$; 2) external heating only is due to forced convection characterized by the constants h and T_e; and 3) the inner surface of the dome is thermally insulated. The solution then is obtained from the graph (in Ref. 14, p.146) for the modified Biot number (i.e., $T_{sd}=T_{sa}$)

$$N_B = q_a \delta / k_p (T_e - T_{sd}) \tag{14}$$

The tradeoff between k_p and δ which is typical for high-temperature-resistant polymers is given in Table 2 for $M=3$ at the altitude of 5000 ft.

Table 2 gives the time it takes the inner surface of a typical high-temperature-resistant polymer dome to reach a specified melt temperature ($T_{sd}=1080°$R) as a function of k_p and δ. A value of $k_p=0.1$ Btu/ft-hr-°R is readily obtained for off-the-shelf high-temperature-resistant polymers. A value of $k_p=0.1$ Btu/ft-hr-°R can be obtained from gas-filled, high-temperature-resistant polymers. A value of $k_p=0.001$ Btu/ft-hr-°R is state-of-the-art for superinsulators. The calculated failure times are conservative and are shorter then expected. The reasons for believing that the failure times are conservative are the following.

1. Extreme thermal loading occurs only at the blunt end of the dome. However, the stresses are at a minimum in this region because of internal pressurization. Hence, the given time does not necessarily indicate failure time, but only the time it takes the inner surface of the dome to reach 1080°R.
2. Heat flow parallel to the shell away from the blunt end of the dome was neglected.
3. Convective heat transfer to the gas within the dome was neglected.
4. Pyrolysis and melting phenomena at the outer surface of the dome was neglected.
5. It was assumed that the dome accelerated to Mach 3 instantaneously.
6. Radiation cooling of the outside surface was neglected.
7. It was assumed that the missile flight was at zero angle-of-attack, yaw, and pitch at all times, hence concentrating the maximum heat generation to the apex.

Intrinsic to the pressurized polymer dome analysis is the assumption that thermal stresses are negligible if the viscoelastic response of candidate polymers is chemically tailored for a given mission. In this manner, the heat that enters the dome is converted into creep phenomena instead of being stored as elastic strain energy. Because of this mechanism, polymers may survive a longer time in transient heating environments than ceramics. A merit of inverse design methodology is that it predicts the viscoelastic and thermal properties needed once a mission is specified. It then remains for the polymer chemist to develop a polymer that can meet the predicted material specifications.

Summary and Conclusion

The thermal and mechanical properties for a high-temperature-resistant polymer to be used as a pressurized dome can be specified by use of the inverse design analysis in this study. This analysis predicts that, at Mach 3, velocity at an altitude of 500 ft, a hemispherical dome about 0.5 in. thick can withstand a temperature of 620°F (1080°R) for about 1 min without failure, if its creep modulus $J(t)$ is of the order of 3.3×10^{-6} in.^{2}/lbf, $\tau'=1$ min, and its thermal conductivity (k_p) is about 0.01 Btu/ft-hr-°R. Longer mission times of about 10 min can be achieved with yet to be identified superinsulation polymers ($k_p=0.001$ Btu/ft-hr-°R). Since experimental data are not available in the required time and temperature domain, it is not known whether such thermal and mechanical properties are yet attainable with some of the new high-temperature heat-resistant polymers.

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